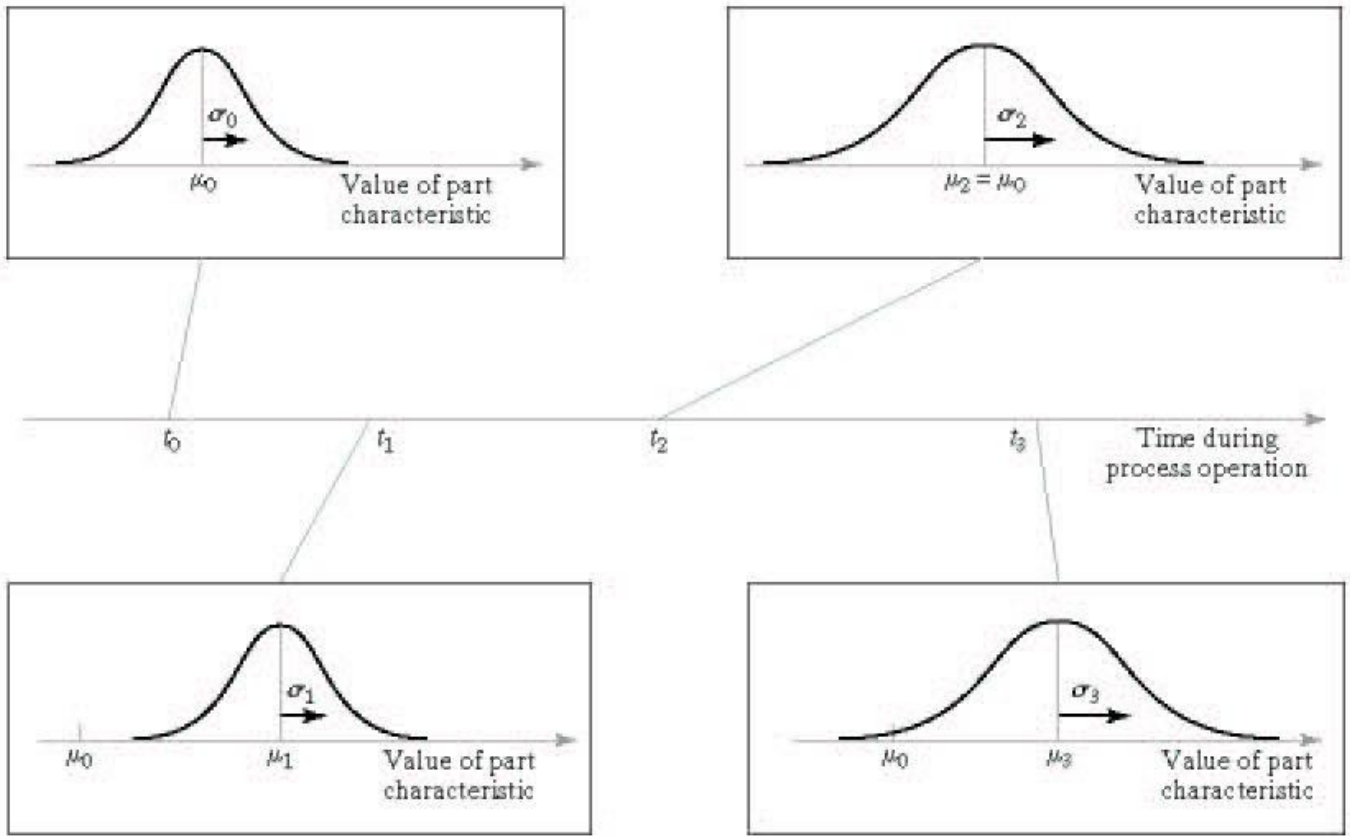
جلسه سیزدهم و چهاردهم: کنترل آماری فرآیند تولید

Process Variability

Manufacturing process variations are of two types:

1. **Random variations** – result from intrinsic variability in the process
   - Process is operating normally
   - Human variations from cycle to cycle, minor variations in starting materials, machine vibration
2. **Assignable variations** – indicate an exception from normal operating conditions
   - Operator errors, defective raw materials, tool failures, equipment malfunctions

Distribution of Values of Part Characteristics at Four Times during process operation



Process Capability

$$PC = \mu \pm 3\sigma$$

where *PC* = process capability, μ = process mean set at nominal value of the parameter of interest (bilateral tolerances assumed), σ = standard deviation of the process

- Assumptions:
  - Output is normally distributed
  - Steady state operation
  - Process is in statistical control

Process Capability and Tolerances

- Natural tolerance limits – when tolerance is set = process capability
- Process capability index

$$PCI\ (Cp) = \frac{UTL - LTL}{6\sigma}$$

where *PCI* = process capability index, *UTL* and *LTL* = upper and lower tolerance limits, and 6σ = range of natural tolerance limits

Process Capability Index

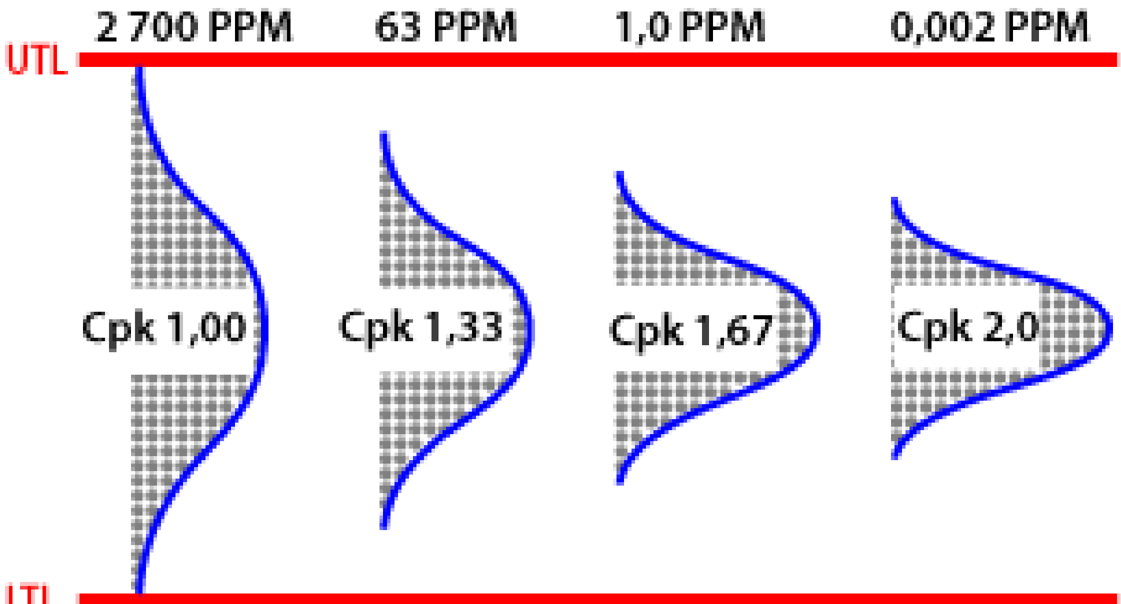


Fig 12

$$C_p = \frac{USL - LSL}{6 \times \hat{\sigma}}$$

$$c_{pk} = \min\left(\frac{USL - \mu}{3 \times \hat{\sigma}}, \frac{\mu - LSL}{3 \times \hat{\sigma}}\right)$$

Cp and Cpk

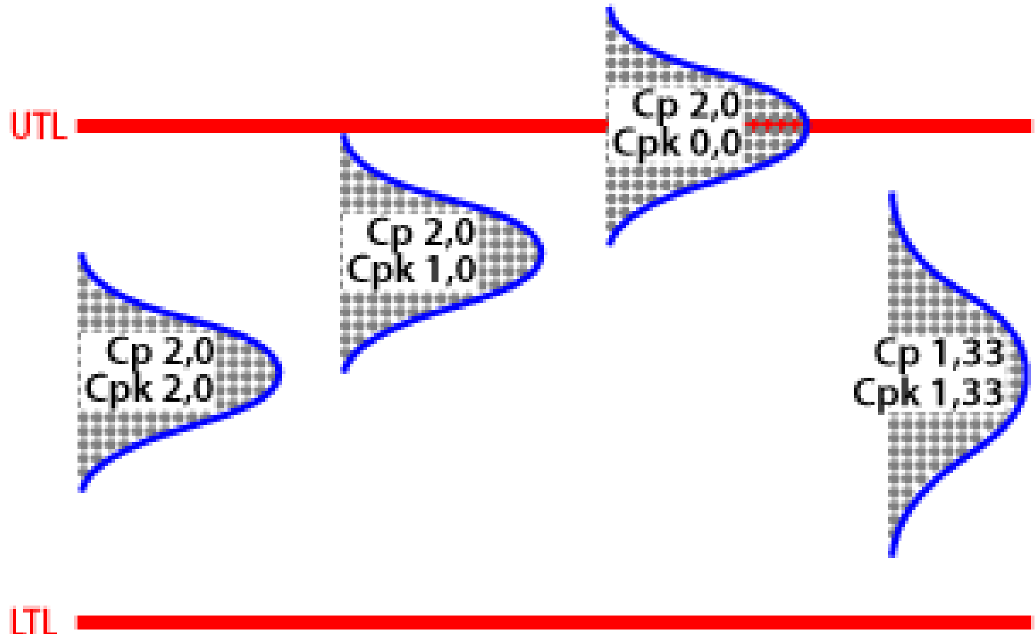


Fig 4

Process Capability Index

Process Capability Index (PCI)	Tolerance = Number of Standard Deviations	Defect Rate (%)	Defective Parts per Million	Comments
0.333	±1.0	31.74	317,400	Sortation required.
0.667	±2.0	4.56	45,600	Sortation required.
1.000	±3.0	0.27	2,700	Tolerance = process capability.
1.333	±4.0	0.0063	63	Significant reduction in defects.
1.667	±5.0	0.000057	0.57	Rare occurrence of defects.
2.000	±6.0	0.0000002	0.002	Defects almost never occur.

Statistical Process Control (SPC)

Use of various methods to measure and analyze a process, either in manufacturing or non-manufacturing situations

- Objectives of SPC:
  1. Improve quality of process output
  2. Reduce process variability and achieve process stability
  3. Solve processing problems

Seven Tools of SPC

Sometimes referred to as the "magnificent seven"

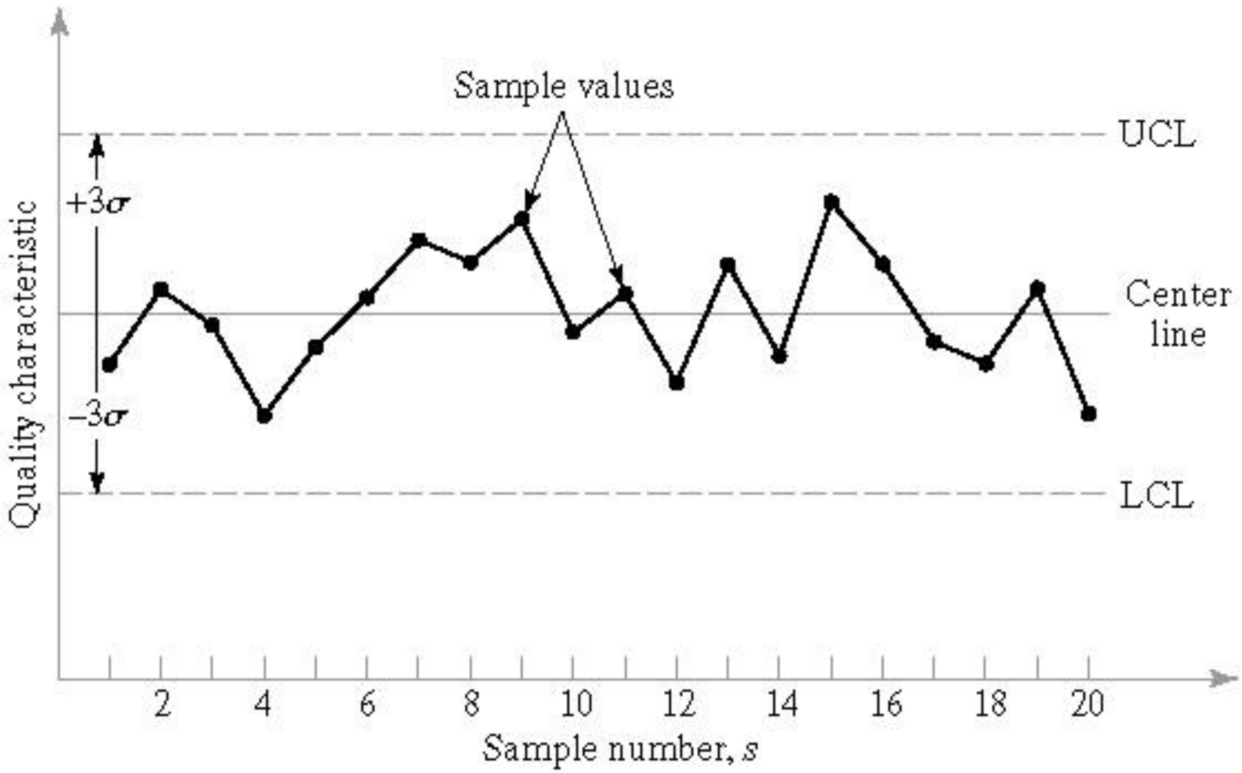
1. Control charts
2. Histograms
3. Pareto charts
4. Check sheets
5. Defect concentration diagrams
6. Scatter diagrams
7. Cause and effect diagrams

Control Charts

A graphical technique in which statistics computed from measured values of a process characteristic are plotted over time to determine if the process remains in statistical control

- Underlying principle is that the variations in a process divide into two categories:
  1. Random variations
  2. Assignable variations

Control Chart



Two Basic Types of Control Charts

1. Control charts for variables
   - Require a measurement of the quality characteristic of interest
   - Two principle types: (1) *X*-bar chart and (2) *R* chart
2. Control charts for attributes
   - Require a determination of either fraction of defects in the sample or number of defects in the sample
   - Two principle types: (1) *p* chart and (2) *c* chart

Determination of LCL and UCL

$$LCL = \bar{\bar{x}} - A_2\bar{R}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$

$$LCL = D_3\bar{R}$$

$$UCL = D_4\bar{R}$$

TABLE 21.2 Constants for the $\bar{x}$ and R Charts

Sample Size n	$\bar{x}$ *Chart* A_2	*R Chart* D_3	D_4
3	1.023	0	2.574
4	0.729	0	2.282
5	0.577	0	2.114
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777

Example

Although 20 or more samples are recommended, let us use a much smaller number here to illustrate the calculations. Suppose eight samples ($m = 8$) of size 5 ($n = 5$) have been collected from a manufacturing process that is in statistical control, and the dimension of interest has been measured for each part. It is desired to determine the values of the center, LCL, and UCL to construct the $\bar{x}$ and R charts. The calculated values of $\bar{x}$ and R for each sample are given below (measured values are in centimeters), which is step (1) in our procedure.

s	1	2	3	4	5	6	7	8
$\bar{x}$	2.008	1.998	1.993	2.002	2.001	1.995	2.004	1.999
R	0.027	0.011	0.017	0.009	0.014	0.020	0.024	0.018

Solution

Solution: In step (2), we compute the grand mean of the sample averages.

$$\bar{\bar{x}} = \frac{2.008 + 1.998 + 1.993 + 2.002 + 2.001 + 1.995 + 2.004 + 1.999}{8} = 2.000 \text{ cm}$$

In step (3), the mean value of R is computed.

$$\bar{R} = \frac{0.027 + 0.011 + 0.017 + 0.009 + 0.014 + 0.020 + 0.024 + 0.018}{8} = 0.0175 \text{ cm}$$

In step (4), the values of LCL and UCL are determined based on factors in Table 21.2. First, using Eq. (21.6) for the $\bar{x}$ chart,

$$\text{LCL} = 2.000 - 0.577(0.0175) = 1.9899$$

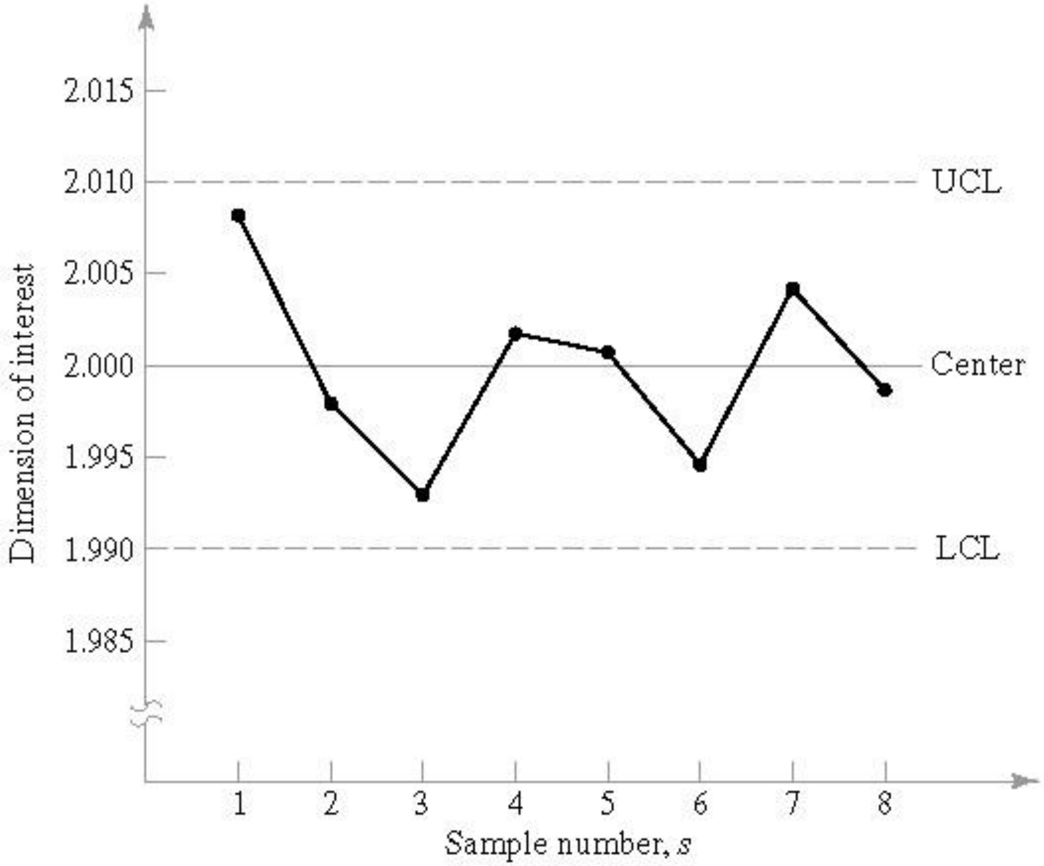
$$\text{UCL} = 2.000 + 0.577(0.0175) = 2.0101$$

And for the R chart using Eq (21.7),

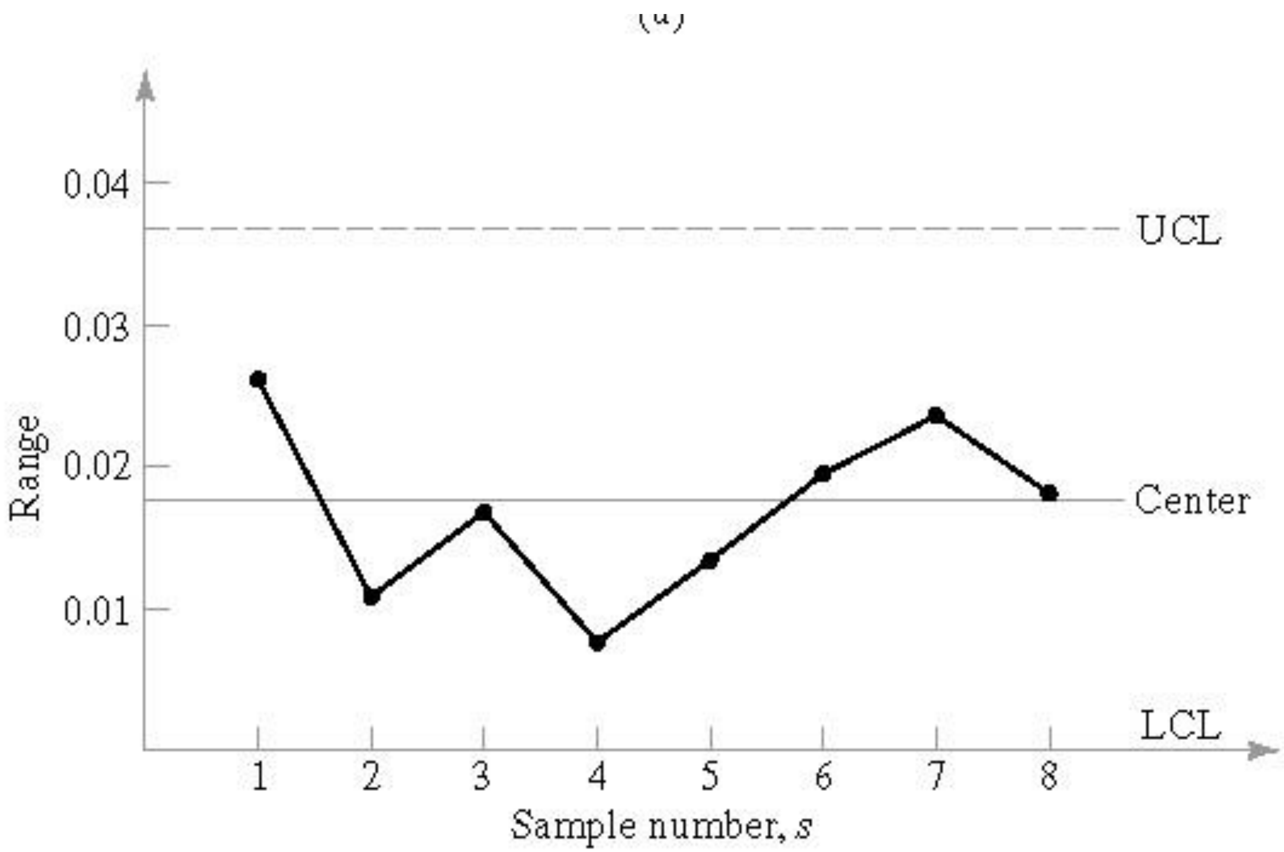
$$\text{LCL} = 0(0.0175) = 0$$

$$\text{UCL} = 2.114(0.0175) = 0.0370$$

X-Bar Chart


2.015
2.010
2.005
2.000
1.995
1.990
1.985
UCL
Center
LCL
Dimension of interest
1
2
3
4
5
6
7
8
Sample number, *s*

R Chart



(a)
0.04
0.03
0.02
0.01
Range
UCL
Center
LCL
1
2
3
4
5
6
7
8
Sample number, *s*

Control Charts for Attributes

- Control charts for attributes monitor the number of defects present in the sample or the fraction defect rate as the planed statistic.

- Examples of these kinds of attributes include number of defects per automobile, fraction of nonconforming parts in a sample, existence or absence of flash in a plastic molding, and number of flaws in a roll of sheet steel. Inspection procedures that involve GO/NO-GO gaging arc included in this group since they determine whether a part is good or bad

Control Charts for Attributes

- **(1) the *p chart***, which plots the fraction defect rate in successive samples; and **(2) the c *chart***, which plots the number of defects, flaws. or other nonconformities per sample

$$p_i = \frac{d_i}{n}$$

$$\bar{p} = \frac{\sum_{i=1}^{m} p_i}{m}$$

the *p chart Formulation*

p chart

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Example

Ten samples ($m = 10$) of 20 parts each ($n = 20$) have been collected. In one sample there were no defects; in three samples there was one defect; in five samples there were two defects; and in one sample there were three defects. Determine the center, LCL, and UCL for the p chart.

Example

Solution: The center value of the control chart can be calculated by summing the total number of defects found in all samples and dividing by the total number of parts sampled:

$$\bar{p} = \frac{1(0) + 3(1) + 5(2) + 1(3)}{10(20)} = \frac{16}{200} = 0.08 = 8\%$$

The LCL is given by Eq. (21.12a):

$$\text{LCL} = 0.08 - 3\sqrt{\frac{0.08(1 - 0.08)}{20}} = 0.08 - 3(0.06066) = 0.08 - 0.182 \rightarrow 0$$

The upper control limit, by Eq. (21.12b):

$$\text{UCL} = 0.08 + 3\sqrt{\frac{0.08(1 - 0.08)}{20}} = 0.08 + 3(0.06066) = 0.08 + 0.182 = 0.262$$

c chart

- c = number of imperfections discovered per 100 m

$$\bar{c} = \frac{\sum_{i=1}^{m} c_i}{m}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

Example

A continuous plastic extrusion process is considered to be operating in statistical control, and it is desired to develop a c chart to monitor the process. Eight hundred meters of the extrudate have been examined and a total of 14 surface defects have been detected in that length. Develop the c chart for the process, using defects per hundred meters as the quality characteristic of interest.

Solution: The average value of the parameter c can be determined using Eq. (21.14):

$$\bar{c} = \frac{14}{8} = 1.75$$

This will be used as the center for the control chart. The LCL is given by Eq. (15a):

$$\text{LCL} = 1.75 - 3\sqrt{1.75} = 1.75 - 3(1.323) = 1.75 - 3.969 \rightarrow 0$$

And the UCL, using Eq. (21.15b):

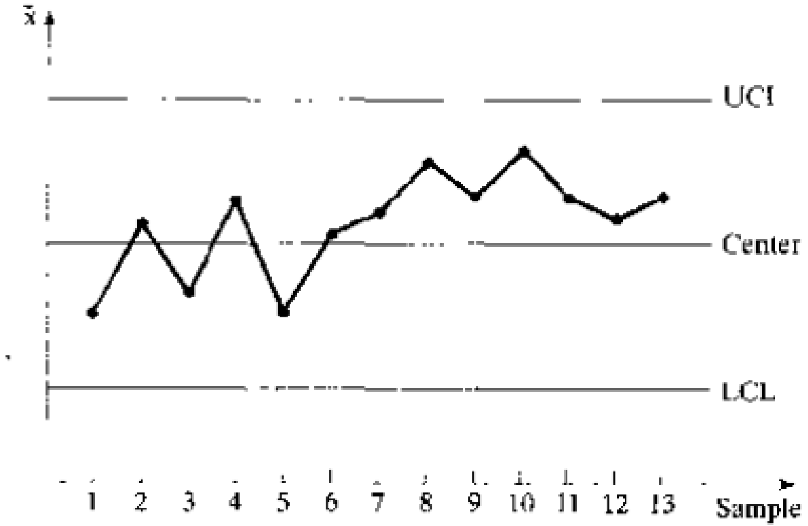
$$\text{UCL} = 1.75 + 3\sqrt{1.75} = 1.75 + 3(1.323) = 1.75 + 3.969 = 5.719$$

Interpreting the Control Charts

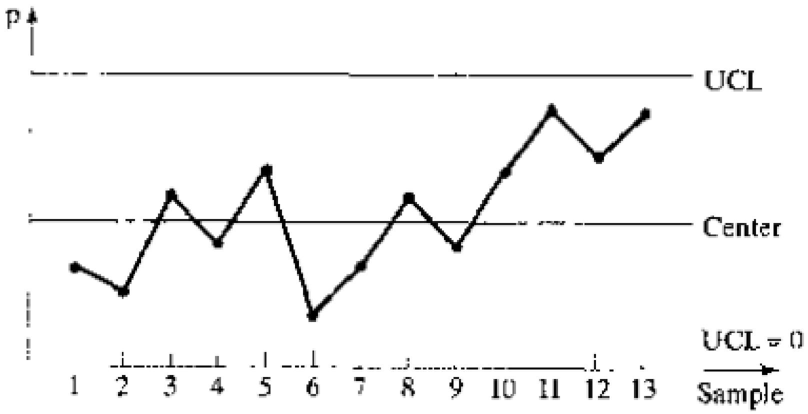
- one point that lies outside the UCL or LCL
- two out of three consecutive points that lie beyond $\pm 2\sigma$ on one side of the center line of the control chart
- four out of five consecutive points that lie beyond $\pm 1\sigma$ on one side of the center line of the control chart
- eight consecutive points that lie on one side of the center line
- six consecutive points in which each point is always higher than its predecessor or six consecutive points in which each point is always lower than its predecessor.

Example

In each chart can you find out of control item in the process?

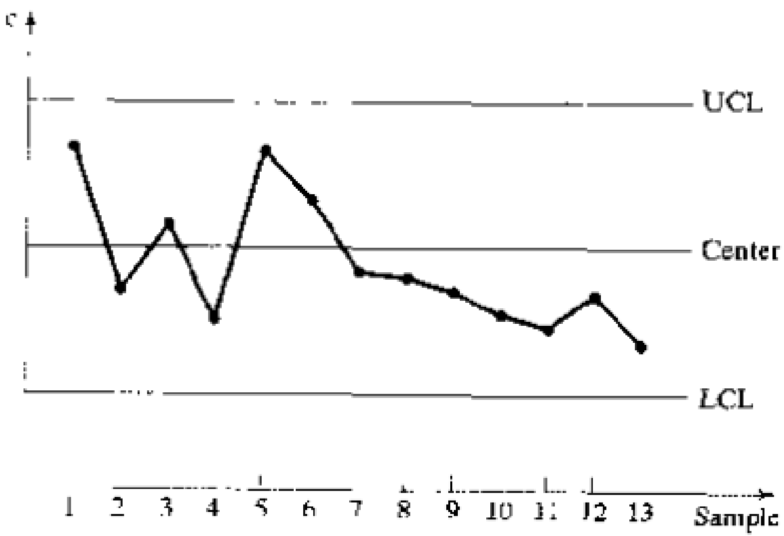


(a)

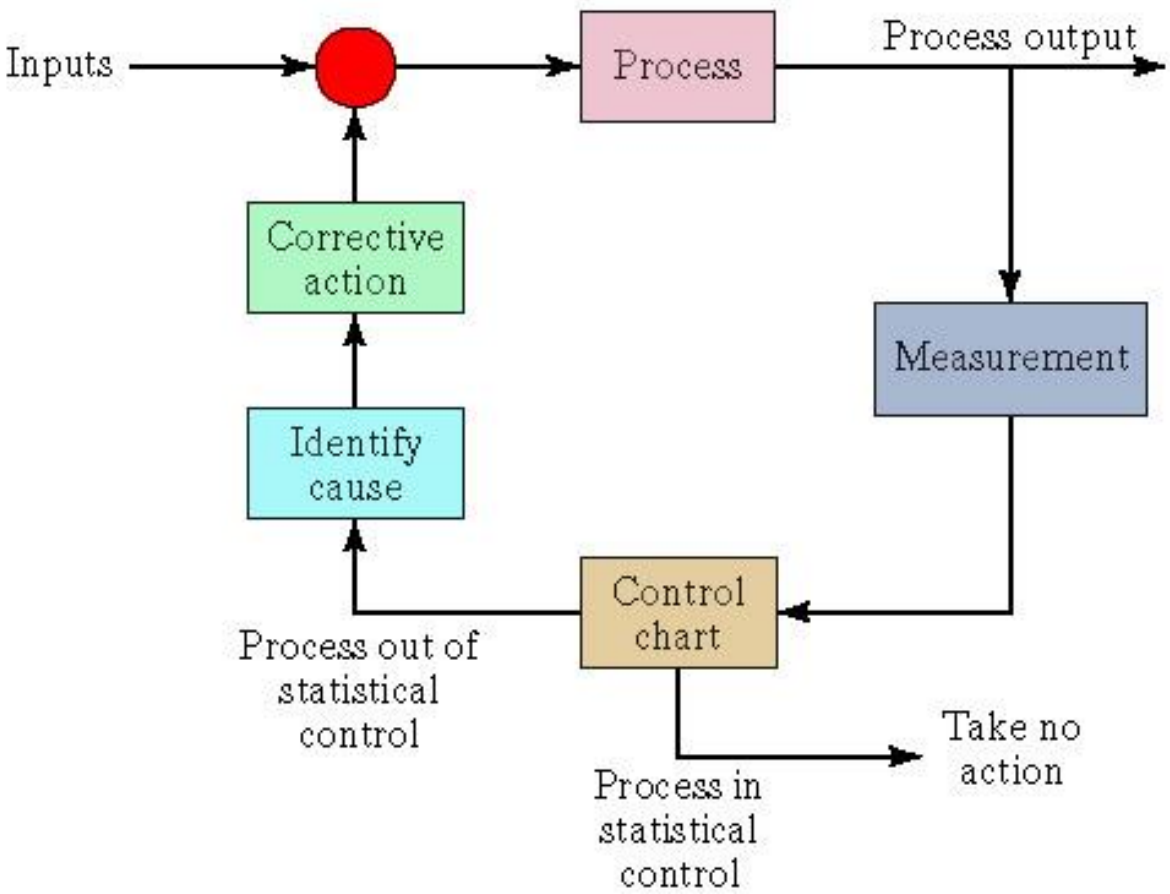


(b)

Example



Control Chart used as Feedback Loop in Statistical Process Control

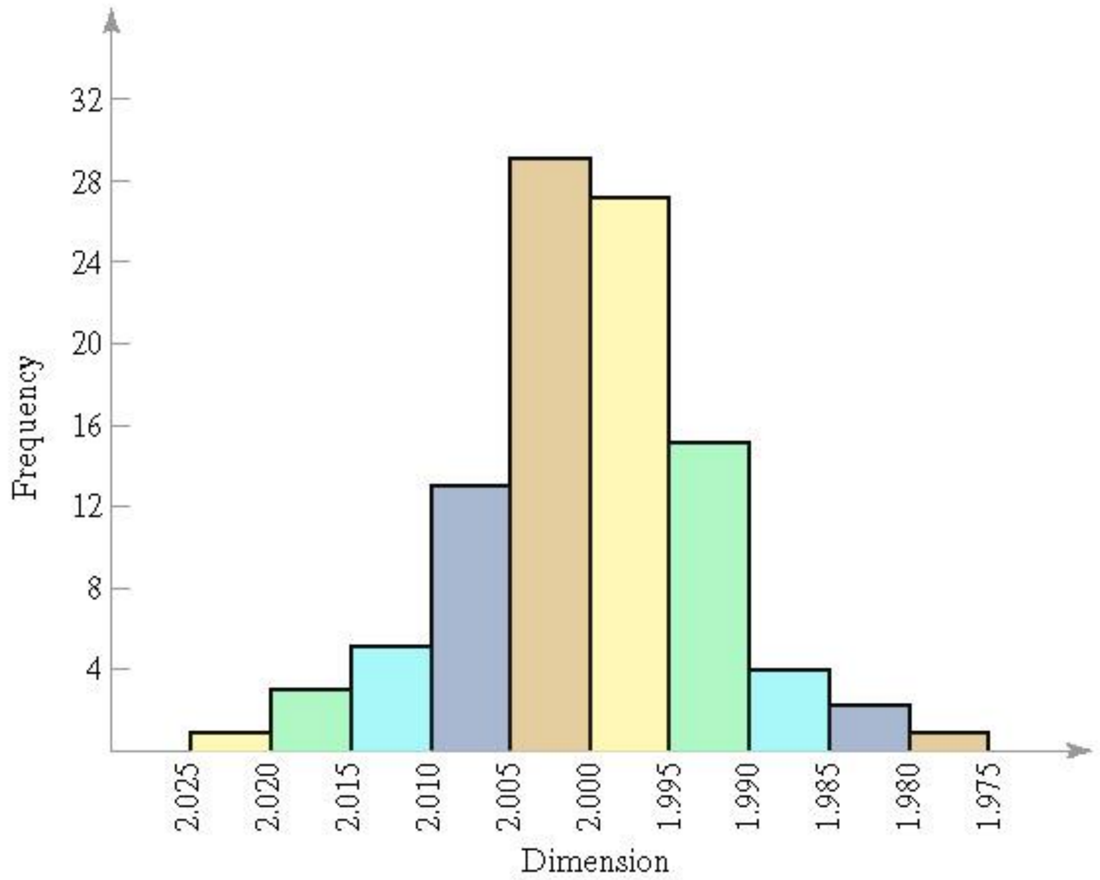


Histogram

Statistical graph consisting of bars representing different members of a population, in which the length of each bar indicates the frequency or relative frequency of each member

- A useful tool because the analyst can quickly visualize the features of the data, such as:
  - Shape of the distribution
  - Any central tendency in the distribution
  - Approximations of the mean and mode
  - Amount of scatter in the data

Histogram

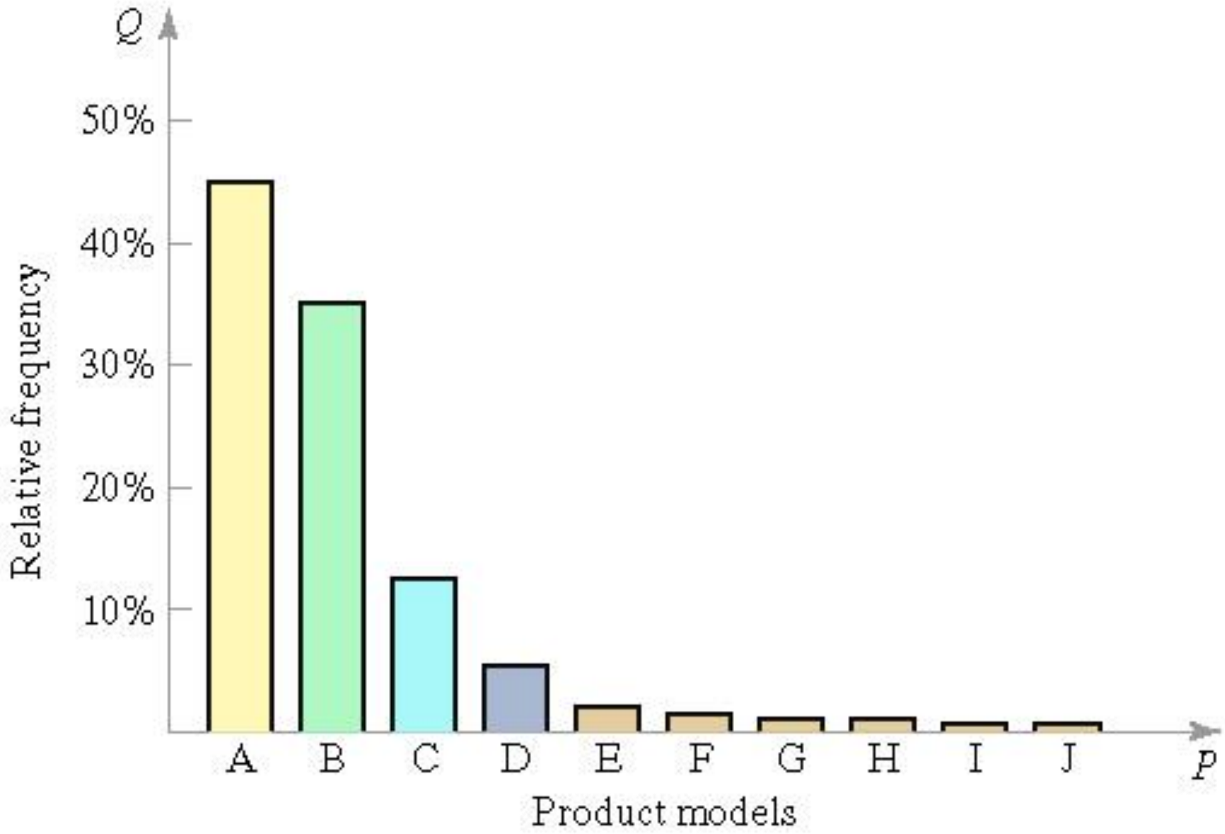

Frequency
32
28
24
20
16
12
8
4
2.025
2.020
2.015
2.010
2.005
2.000
1.995
1.990
1.985
1.980
1.975
Dimension

Pareto Chart

Special form of histogram in which attribute data are arranged according to some criterion such as cost or value

- Based on Pareto’s Law: “the vital few and the trivial many”
- Often identified as the 80%-20% rule
  - 80% of a nation’s wealth is owned by 20% of the population

Pareto Chart



Check Sheet

Data collection tool generally used in the preliminary stages of a study of a quality problem

- Data often entered by worker as check marks in a given category
- Examples:
  - Process distribution check sheet - data on process variability
  - Defective item check sheet – types and frequencies of defects on the product
  - Defect location check sheet - where defects occur on the product

Check sheet- Example

تعداد قطات تولید شده در محدوده های اندازه ای مختلف در سه شیفت ۱، ۲ و ۳

Range of Dimension	*May 5*	*May 6*	*May 7*	*May 8*	*May 9*	*Weekly Totals*
1.975 ≤ x < 1.980			3			1
1.980 ≤ x < 1.985		2		3	3	3
1.985 ≤ x < 1.990	1	3	3	1	3	5
1.990 ≤ x < 1.995	1 2	11 2 3	1 2	1 2	1 22	13
1.995 ≤ x < 2.000	11 222 3	11 22 3	111 222 3	11 222 3	11 22 3	29
2.000 ≤ x < 2.005	11 22 3	11 22 3	111 22 3	11 222 3	111 22	27
2.005 ≤ x < 2.010	1 2 3	1 2 3	22 3	1 33	1 2 3	15
2.010 ≤ x < 2.015	3	3	3		3	4
2.015 ≤ x < 2.020	3			3		2
2.020 ≤ x < 2.025	3					1
Total Parts/Day	20	20	21	20	19	100

Check sheet- Example

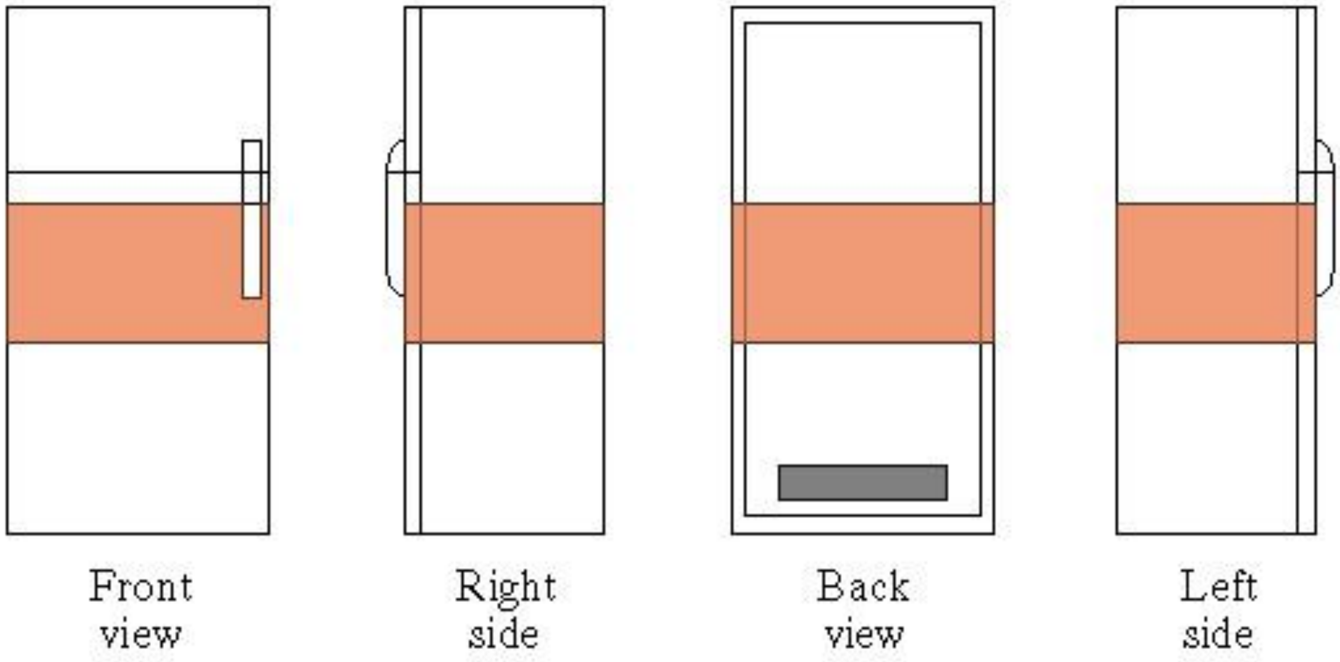
Range of Dimension	Shift 1	Shift 2	Shift 3	Totals
$1.975 \leq x < 1.980$			1	1
$1.980 \leq x < 1.985$		1	2	3
$1.985 \leq x < 1.990$	2		3	5
$1.990 \leq x < 1.995$	6	6	1	13
$1.995 \leq x < 2.000$	11	13	5	29
$2.000 \leq x < 2.005$	12	11	4	27
$2.005 \leq x < 2.010$	4	5	6	15
$2.010 \leq x < 2.015$			4	4
$2.015 \leq x < 2.020$			2	2
$2.020 \leq x < 2.025$			1	1
Weekly Total Parts/Shift	35	36	29	100
Average Daily Parts/Shift	7.0	7.2	5.8	

Defect Concentration Diagram

A drawing of the product (all relevant views), onto which the locations and frequencies of various defect types are added

- Useful for analyzing the causes of product or part defects
- By analyzing the defect types and corresponding locations, the underlying causes of the defects can possibly be identified

Defect Concentration Diagram



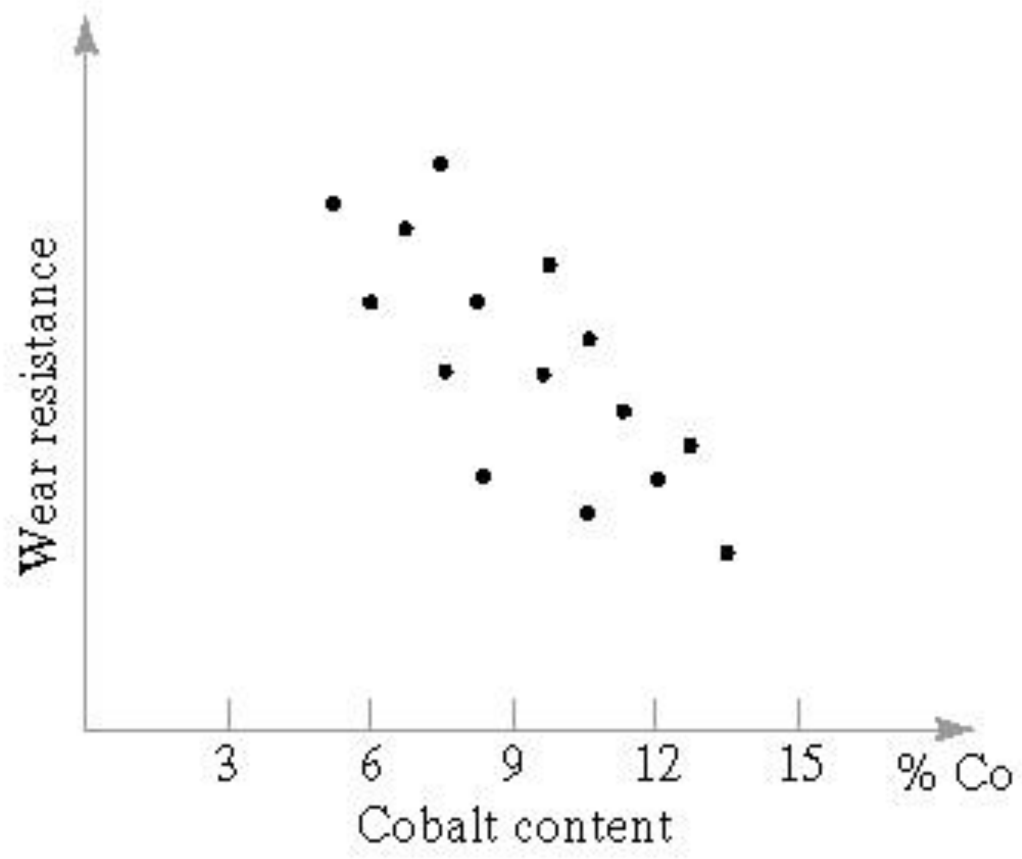
Four views of refrigerator with locations of surface defects indicated in colored areas

Scatter Diagrams

An x-y plot of data collected on two variables, where a correlation between the variables is suspected

- The data are plotted as pairs; for each x_i value, there is a corresponding y_i value
- The shape of the collection of data points often reveals a pattern or relationship between the two variables

Effect of cobalt binder content on wear resistance of a cemented carbide cutting tool insert

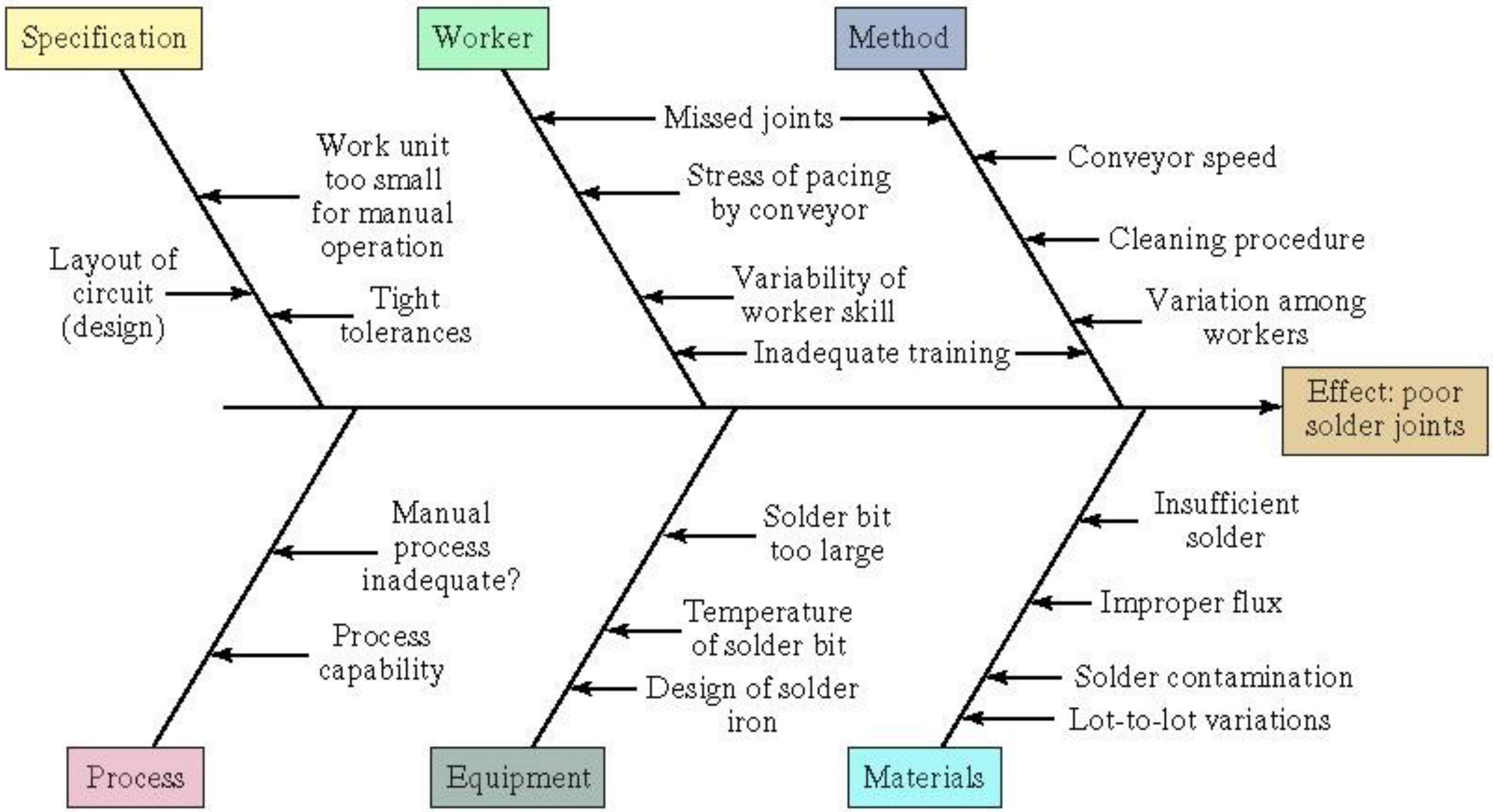


Cause and Effect Diagram

A graphical-tabular chart used to list and analyze the potential causes of a given problem

- Also known as a “fishbone diagram”
- Can be used to identify which causes are most consequential and how to take corrective action against them

Cause-and-Effect Diagram



Applications of the Seven SPC Tools in a Quality Improvement Project

Quality Improvement Project Step	*SPC Tool*	*Other Techniques and Actions*
1. Select the project	Control charts Pareto chart	Pareto priority index
2. Observe the process	Check sheet	Check list Propose theories and hypotheses
3. Analyze the project	Histogram Pareto chart Defect concentration diagram Scatter diagram Cause and effect diagram	Conduct experiments Computer simulations Evolutionary operations on actual process Literature review
4. Formulate corrective action	Scatter diagram Cause and effect diagram	Make recommendations Management approval and authorization
5. Implement corrective action		Revise procedures Manage change Project assessment (audit) Disband team

Pareto priority index

$$PPI = \frac{E(S)}{CT}$$

E(S) = expected savings from the project. which equals anticipated savings multiplied by the probability of success; C = cost of project: and *T* = time to complete (yr).

Example

Six quality improvement projects are being considered for possible selection. The anticipated project cost, savings, probability of success, and time to complete are given in the accompanying table. Which project should be selected if the Pareto priority index is used as the selection criterion?

Project	*Cost*	*Savings*	*Pr(Success)*	*Time to Complete*
A	$20,000	$50,000	0.80	1.5 yr
B	$10,000	$34,000	0.90	1.2 yr
C	$35,000	$60,000	0.75	2.0 yr
D	$6,000	$25,000	0.90	1.5 yr
E	$25,000	$90,000	0.60	2.5 yr
F	$20,000	$80,000	0.85	0.75 yr

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